B028
Predictive Deconvolution for Correction of Nonstationary Seismic Records
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SUMMARY
Generalization of the autoregressive model for the case of nonstationarity is used. Proposed way of parameterization of nonstationarity allows effective correction of the waveform at even significantly nonstationary seismic data. Nonstationary predictive deconvolution described can be used both for elimination of reverberations and for wavelet correction and provides better results than the traditional approach.
When seismic record does not satisfy stationary convolutional model the problem of waveform correction in seismic data processing is of critical importance. Nonstationarity of seismic record is caused by several reasons. Three of them are most significant:

1. Geometrical divergence.
2. Frequency-dependent attenuation, caused by absorption
3. Presence of coherent noise, that differs from the signal by frequency content.

During coherent noise elimination, single-channel convolutional model gets broken and frequency content of the signal is changed nonstationarily depending on time.

Non-elastic absorption traditionally is considered to be the main reason for waveform nonstationarity.

The above mentioned reasons make one to try and develop techniques of the waveform correction by time-varying filters.

Among possible solutions of the waveform correction problem, prediction algorithms, i.e. deconvolutions using autoregressive model in one way or another, are most commonly used. Generalization of the autoregressive model for the case of nonstationarity leads to nonstationary autoregression of a seismic trace \( z(t) \):

\[
z(t) = \sum_k z(t-k)a(t,k) + \xi(t), \quad (1)
\]

where \( t \) – discrete time, and \( \xi(t) \) - generating process corresponding to spikogram.

In this paper, the following way of nonstationarity parameterization, convenient for generation of a seismic model of a time series, is proposed. We assume that autoregression coefficients \( a(t,k) \) can be taken in the following form:

\[
a(t,k) = \sum_{n=0}^{M} (t-k)^{ap} a_n(k),
\]

where \( p \) – some numerical multiplier, and \( a_n(k) \) – expansion coefficients.

It can be proven that this model can describe:

1. trace multiplied by polynomial of integer power not greater than \( M \) (nonstationarity caused by geometrical divergence correction)
2. trace with nonstationarity caused by frequency-dependent absorption with a constant \( Q \).

A trace can be described as following:

\[
z(t) = \sum_n \sum_k z(t-k) \cdot a_n(k) \cdot (t-k)^{ap} + \xi(t).
\]

The following functional can serve as a criterion for estimation of \( a_n(k) \) parameters for one trace:

\[
J = \sum_t (z(t) - \sum_n \sum_k z(t-k) \cdot a_n(k) \cdot (t-k)^{ap})^2 \quad (2).
\]

And at the same time this criterion can be modified for a set of traces for which parameters \( a_n(k) \) are the same on some base of averaging. For practical usage it is extremely important that it is possible to write this criterion for the situation of a limited bandwidth.

Note that nonstationary model for trace (1) also can be written in frequency domain:

\[
z(t) = \int_{\omega_1}^{\omega_2} \left[ A(t,\omega) \cdot Z(\omega) + \xi(\omega) \right] \cdot e^{j\omega t} d\omega, \quad (3)
\]

\( A(t,\omega) \) allows expansion:

\[
A(t,\omega) = \sum_n A_n(\omega) \cdot t^n
\]

with \( A(t,\omega) = 0 \) outside of the range \( (\omega_1, \omega_2) \).
To prove the equivalence or representations (1) and (3) one should use the behaviour of convolution:

\[ (a(t) \ast b(t)) \cdot t^n = \sum_{k=1}^{n-1} (a(t) \cdot t^{n-k}) \ast (b(t) \cdot t^k) \cdot C_n^{k+1}, \]

where \( C_n^k \) - binomial coefficients.

For \( A(t, \omega) \) operator being an autoregression (prediction) operator, it is required that it is casual (i.e. its real and imaginary parts are interrelated by Hilbert transform) and that

\[ \int_{\omega_1}^{\omega_2} A(t, \omega) d\omega = \int_{\omega_1}^{\omega_2} A(t, \omega) d\omega = 0. \]

This implies that the operator often being used for attenuation correction (Gelius 1987) is a special case of nonstationary autoregression operator (1) and (3), accurate within normalizing function \( e^{c(t)} \), where \( c(t) = -\frac{1}{\omega_2 - \omega_1} \int_{\omega_1}^{\omega_2} P(\omega) d\omega \), where \( \rho(t) \) describes smooth function of attenuation decrement.

In fact this operator transforms signal to stationary form only if \( \rho(t) = \text{const} \), that is attenuation decrement is constant, but if decrement variation is smooth fluctuations of the wave form after correction are small.

If we denote \( z(t) \cdot t^{p_m} = q_m(t) \), then functional (2) can be rewritten in following form

\[ J = \sum_t (z(t) - \sum_m \sum_k q_m(t-k) \cdot a_m(k))^2 \]  

(4)

In such formulation the expression (4) represents the common problem of multichannel filtration. Derivation of (4) by \( a_m(k) \) leads to system of equations with Block-Toeplitz matrix. Effective methods of solving these systems are well explored (Wiggins and Robinson 1978).

Obviously \( q_m(t) \) forms basis generated by single trace \( z(t) \) and this basis allows describing nonstationarity. The number of parameters \( a_m(k) \) that have to be estimated depends on how one chooses the basis.

For instance, if we choose

\[ q_m(t) = \left( \sum_j z(i-j) \cdot c_m(j) \right) \cdot t^{p_m}, \]

where \( c_m(j) \) - predetermined “good” autoregression parameters, then the number of estimated parameters \( a_m(k) \) can be considerably decreased. If nonstationarity is described by frequency dependent attenuation function, than \( c_m(k) \) are in fact autoconvolutions of cepstrum of attenuation operator. The number of estimated parameters can be also decreased by preliminary stationary filtration.

Obviously, the functional (4) can be averaged over a set of traces and it is very important for processing of real records. Though of course adjacent traces are not independent realizations of the stochastic process, such an averaging protects the algorithm from influence of particular realization of a spikogram on filtration result.

Below, an example of nonstationary deconvolution results is provided.

Figure 1A shows a model time-section of reflectivity factors bandpass filtered with the band of 5-60 Hz. Then the attenuation was introduced and the result was convolved with a minimum-phase wavelet to obtain synthetic data for the tests (Figure 1B). Amplitude spectrum of the signal had one clearly defined maximum at 18 Hz and rapidly damped with frequency. The model time-section consists of several packs of reflections.
Figure 2A shows the result of traditional predictive deconvolution of the synthetic data, Figure 2B – the result of nonstationary predictive deconvolution with using of four power functions. At such a scale the difference is not very obvious, however it is clearly seen that the main changes are at the bottom of the time-section.

Figure 3 shows the same results but only the bottom part of the time-section. We can see that usual deconvolution failed in wave resolution (Figure 3A) while the result of the nonstationary deconvolution (Figure 3B) is relatively close to the ideal result.

Figure 4A. Figure 4B shows the result of nonstationary predictive deconvolution. At the bottom centre of Figures 3 and 4 amplitude spectra calculated for selected fragments of time-sections are shown.

For comparison, the ideal result (i.e. synthetics without attenuation) is represented on Figure 4A. Figure 4B shows the result of nonstationary predictive deconvolution. At the bottom centre of Figures 3 and 4 amplitude spectra calculated for selected fragments of time-sections are shown.
Furthermore, the procedure turned out to be efficient in elimination of reverberations. In general, the problem of reverberation elimination is equal to the problem of multiples suppression, requiring correct modeling of the multiples. But in case of shallow water multiple suppression and deconvolution problems become similar. And if for multiple modeling it is necessary to take proper account of time shifts depending on angle of incidence and bottom configuration, for deconvolution the main problem is time-dependant variability of reverberation period. The proposed nonstationary deconvolution can be taken as a possible solution of this problem.

Figure 3. Detailed view on the bottom part of Figure 2, example of (A) predictive and (B) nonstationary predictive deconvolution results, with the amplitude spectra plots of both of them.

Figure 4. Comparison of (A) model (bandpassed spikogramm) and (B) result of nonstationary predictive deconvolution, with the amplitude spectra plots of both of them.

The algorithm has also been tested on the real seismic data and demonstrated good results both in waveform correction and elimination of reverberations. Note, that applying of
nonstationary deconvolution the signal amplitudes remain nonstationary. We used various ways of amplitude adjustment, but most correct one would be available if one identify nonstationarity parameters. Thus we suspect that if one assume that all nonstationarity of the reflections is due to frequency-dependant attenuation, the parameters describing this attenuation (attenuation decrement or Q) can be determined from the operators used to corrected this effect.