

# Efficient multiple attenuation with a dealiased velocity filtering in f-x domain

Mikhail Denisov and Dmitri Finikov  
117071, Geotechsystems, Ordzhonikidze, 13/2, Moscow, Russia

## *Introduction*

A procedure for transforming data from the original source/receiver depth level to a different one is usually called wavefield extrapolation. The most important applications of this technique are simulation of multiples and peg-legs for their subsequent subtraction and pre- or post-stack datuming.

A typical approach to the design of the wavefield extrapolator is the use of the wave equation that results in a Kirchhoff summation operator that maps the surface data into a multiple model, or, in other words, predict the multiples (Berryhill and Kim, 1986). Datuming, employing the wave-equation method uses the Kirchhoff operator to move the recorded wavefield to a different acquisition plane (Beryhill, 1979 and Beryhill, 1984). One of the most substantial disadvantages of such wavefield extrapolation scheme are inappropriate stacking conditions in the region of tangency. Spatial aliasing is a common problem with the performance of such operators. Sometimes it can cause strong artifacts that degrade the quality of the output.

A possible way to attack aliasing is to design a frequency-selective Kirchhoff operator (Gray, 1992). With no a priori information on the actual moveouts this method is usually inefficient. Besides, the dip filter it applies attenuates the high-frequency components of the signal at large dips.

In case a priori information about the actual dips is available the aliasing problem can be well treated with fine aperture tuning.

An improvement of the procedure may be obtained by application of an equivalent of the Kirchhoff operator realized in the tau-p domain. It allows more signals contribute the constructive sum obtained in the point of tangency. The conventional plane-wave decomposition is global, hence the aliasing problem is encountered again.

Some attempts to localize the data summation gate using anti-alias windowing were made (Schultz and Claerbout, 1978). Such approach requires good knowledge of stacking velocities and adequacy of the hyperbolic moveout assumption. In case of conflicting dips this method cannot provide a reliable protection against aliasing.

## *Method*

Our approach in addressing the problem of wavefield extrapolator design is based on combining the advantages of the conventional Kirchhoff operator and the scheme that takes the advantage of the plane-wave decomposition. The drawbacks of these methods are as follows. The former can cause strong signal distortions since the number of signals that contribute to the constructive slant sum is small. The latter provides better summation conditions in the region of tangency, but the sum can be distorted since global summation leads to stacking across events.

To improve the summation conditions, we suggest application of local slant stacking in the tangency point along the tangent line with subsequent integration (summation) along the operator trajectory. In other words, we apply local rather than global summation. In most cases it makes it possible to avoid summation across events that is inevitable in the conventional tau-p

scheme, hence less spurious energy will be produced. Furthermore, this approach has got all the advantages of plane-wave decomposition-based methods since the improved conditions of signal summation remain.

Let  $w_{z_1}(x_s, x_r, t)$  be a trace of the input data recorded at level  $z_1$ . Let  $w_{z_2}(x_s, x_r, t)$  stand for a trace of the data with receiver point mathematically moved to a specified depth level  $z_2$ . The general form of the Kirchhoff transformation for datuming of the receivers can be written as

$$w_{z_2}(x_s, x_r, t) = \left[ \sum_{x=-M}^M w_{z_1}(x_s, x_r - x, t - \tau(x_s, x_r, x)) a(x_s, x_r, x) \right] * f(t), \quad (1)$$

where  $\tau(x_s, x_r, x)$  is the wavefield extrapolation operator trajectory,  $(-M, M)$  is aperture,  $a(x_s, x_r, x)$  is the weight function,  $f(t)$  is a filter for correction of signal distortions. Asterisk stands for convolution.

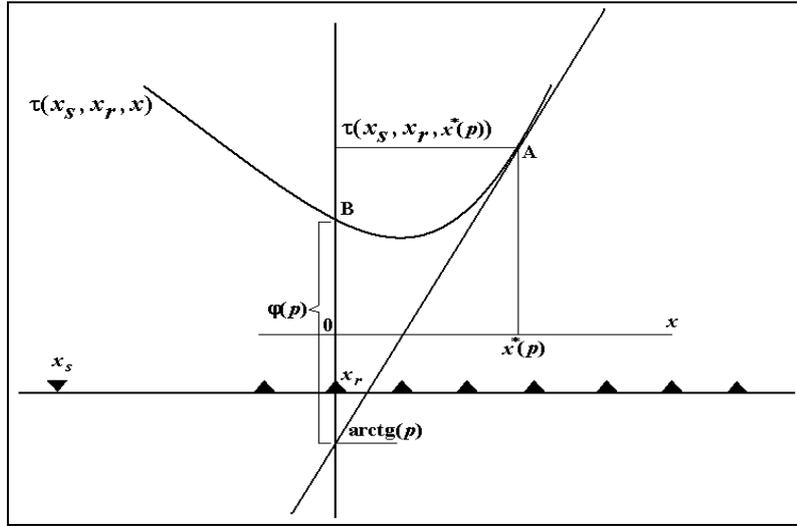


Fig.1: General principles of the conventional data extrapolation schemes.

Since the transformation (1) kinematically moves the signal from point A to point B (see Figure 1), it introduces a shift  $\varphi(p) = \tau(x_s, x_r, x^*(p)) - px^*(p)$  to a plane wave  $\delta(t - px_r)$  on a shot gather. In other words, an ideal output will be  $\delta(t - px_r - \varphi(p))$ . This point is the basis of the plane-wave decomposition-based data extrapolation schemes with a typical flow-chart as follows. Calculate the tau-p transform of the original data sorted in shot gathers, introduce the shifts for every plane wave, and make the inverse transformation into the  $t - x$  domain.

The suggested modification of the Kirchhoff operator performs local data summation around the tangency point  $x^*(p)$  within aperture  $(-L, L)$  along the tangent line  $k(x)$  followed by summation of the results along the operator trajectory. Figure 2 illustrates this principle.

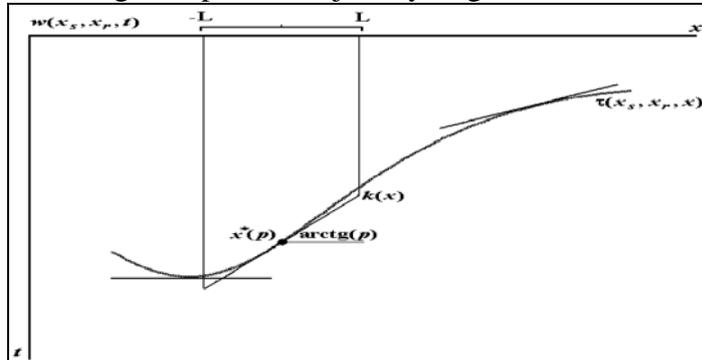


Fig. 2: Operator trajectory and tangent lines for each point  $x^*(p)$  on it. The sloping lines show the trajectory for summation at the point of tangency.

Let  $d(x_s, x^*(p), t)$  denote a local slant sum obtained for a tangency point  $x^*(p)$  along the tangent line  $k(x) = p(x - x^*(p)) + \tau(x_s, x_r, x^*(p))$ :

$$d(x_s, x^*(p), t) = \sum_{x=-L}^L w_{z_1}(x_s, x^*(p) - x, t - k(x + x^*(p))). \quad (2)$$

Then the result of extrapolation of the receivers will be

$$w_{z_2}(x_s, x_r, t) = \int_{-q_1}^{q_2} d(x_s, x^*(p), t) * f_{x^*(p)}(t) dp \quad (3)$$

where, for the sake of convenience, integration along the summation curve dips rather than its  $x$ -coordinates is performed and  $-q_1$ ,  $q_2$  are the minimum and maximum dips of the curve, respectively. The filter  $f_{x^*(p)}(t)$  is applied to compensate for wavelet distortions due to summation.

Let the input data be a plane wave  $w(x_s, x_r, t) = \delta(t - px_r)$ . Having substituted this expression into the frequency domain equivalents of (2) and (3) we obtain

$$W_{z_2}^p(x_s, x_r, \omega) = 2 \int_{-q_1}^{q_2} \frac{\sin((\alpha - p)L\omega)}{(\alpha - p)\omega} e^{-j\omega\psi_p(\alpha)} F_{x^*(\alpha)}(\omega) d\alpha, \quad \psi_p(\alpha) = \tau(x_s, x_r, x^*(\alpha)) - px^*(\alpha)$$

as the ideal output will be

$$\tilde{W}_{z_2}^p(x_s, x_r, \omega) = e^{-j\omega(\psi_p(p) + px_r)}.$$

A possible but a time consuming way to approximate  $\tilde{W}_{z_2}^p(x_s, x_r, \omega)$  by  $W_{z_2}^p(x_s, x_r, \omega)$  is to estimate optimal shaping filters,  $\hat{F}_{x^*(\alpha)}(\omega)$ , using the least squares objective:

$$\hat{F}_{x^*(\alpha)}(\omega) = \arg \min_{\hat{F}_{x^*(\alpha)}(\omega)} \int |\tilde{W}_{z_2}^p(x_s, x_r, \omega) - W_{z_2}^p(x_s, x_r, \omega)|^2 dp.$$

We suggest another approach that is based on local approximation of the summation curve by a parabola. This method is fast and efficient and provides good results. The optimal shaping filters obtained depend only on the curvature of the summation curve in the region of tangency. This property allows calculation of a set of filters before data extrapolation.

#### *Alias protection*

Although the tau-p decomposition involved in the algorithm is local, hence preventing summation across events; the common problem for all 2D filtering techniques is faced again. Dips of the tangent lines that define the summation direction do not necessarily coincide with the actual dips of events. In other words, we have to tackle the aliasing problem.

Our approach is based on an assumption as follows. The low-frequency components of the signal usually are not expected to be aliased after slant summation. On the other hand, aliasing in high frequencies manifests itself in increase of amplitudes for some frequencies at some dips. Clearly, low frequencies are summated best. Therefore, if we specify a band,  $(\omega_1, \omega_2)$ , in low frequencies and estimate the original energy,  $E^{orig}$ , to the output energy,  $E^{out}(\alpha)$ , ratio within the band for every local summation dip,  $\alpha$ , we obtain a threshold,  $R^\alpha$ , that cannot be exceeded at any high frequency. If random noise is present, the energy estimator will produce a biased result. Therefore, the anti-alias algorithm should be equipped with a noise estimator to compensate for the bias. Since the operator design involves the local tau-p decomposition, it is convenient to obtain noise with a similar procedure. We consider the result of local slat summation with weights equal to one (1,1,...,1) to be signal, as an auxiliary sum obtained with weights alternating for adjacent traces (1,-1,...,1) to be noise. Both sums can be calculated recursively. It is an important feature of the algorithm that the signal and noise energy estimation is carried out in a dip-dependent way. This essentially improves the performance of the whole extrapolation scheme.

### Results

A model marine dataset was simulated for testing the local tau-p decomposition based operator with alias protection. The data were mathematically propagated through the water layer to obtain a set of multiple arrivals. The output of the conventional Kirchhoff operator is given in Figure 3 on the left as the result of the new scheme application is on the right.

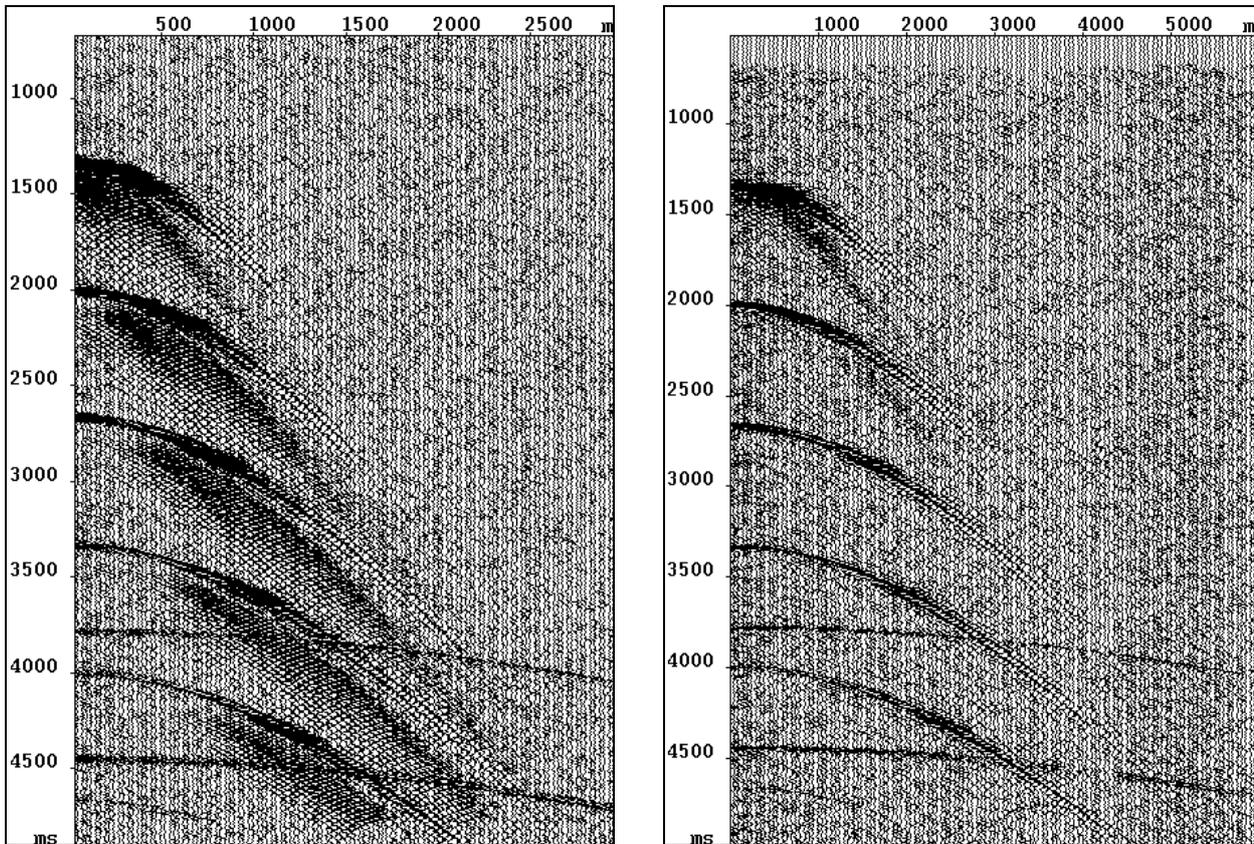


Figure 3. Comparison between the results provided by the conventional forward extrapolator (on the left) and the new one (on the right).

### Conclusions

We have considered only a 2D case, which implies wavefield invariance in the  $y$ -coordinate direction. This is the case for a linear source and earth varying only in  $x$ -coordinate and depth. Consideration of a point source is called a 2.5D problem and its solution leads to different weigh function to account for true amplitudes. The earth is 3D and very seldom (if ever) obeys the assumptions of 2.5D. Therefore, true amplitude processing can hardly be carried out in practice if only a single line is available and that is the reason why we do not work in 2.5D.

This anti-alias scheme suggested shows excellent performance and is fast enough.

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